CHAPTER 2
MEASUREMENTS & CALCULATIONS

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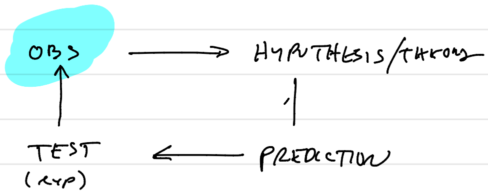
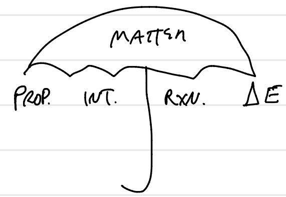




2 Types of Observations

- qualitative
- quantative

Measurements

MEASUREMENT = # + UNIT

accuracy/precision — sci. notation — metric

The convention for a measurement is that the quantity reported should be all known values and the first estimated value

Scientific Notation & The Larger/Smaller Protocol

$a \times 10^n$

$1 \le a < 10$ (put decimal after 1st non-zero digit)

$93{,}000 \times 10^0 = 9.3000 \times 10^4$

$9.3000 \times 10^4 = 93{,}000$ ~~$\times 10^0$~~

5

$10^0 = 1$

602 000 000 000 000 000 000 000

6.02×10^{23}

12

The Larger/Smaller Protocol Illustrated

$$2 \times 4 = \boxed{1} \times 8$$

2x

L

$$0.000167 \times 10^{0} = 1.67 \times 10^{-4}$$

S

Ex2 0.000167×10^{0}

1.67×10^{-4}

Ex3 1.67×10^{-2}

$0.00\,0167 \times 10^{0}$

$$\frac{a}{b} = \frac{c}{de}$$

$$e = \frac{c \cdot b}{d \cdot a}$$

Scientific Notation In Mathematical Operations

Multiplication/Division

$(5.8 \times 10^2)(4.3 \times 10^8)$

$$\begin{array}{r} 5.8 \\ \times\, 4.3 \\ \hline 24.94 \end{array} \times 10^{10}$$

2.494×10^{11}

2.49×10^{11}

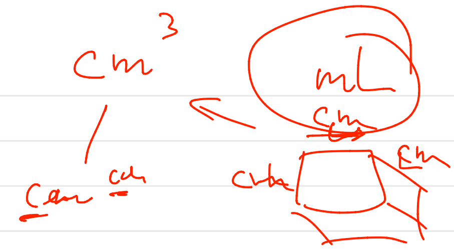
Addition/Subtraction

$$\begin{array}{r} 8.99 \times 10^2 \\ -\ 1.8 \times 10^3 \end{array}$$

$$\begin{array}{r} 0.899 \times 10^3 \\ -1.8 \times 10^3 \\ \hline \times 10^3 \end{array}$$

88.1×10^3

Metric System



MEASUREMENT = # + UNIT

↓

UNIT = PREFIX + BASE UNIT

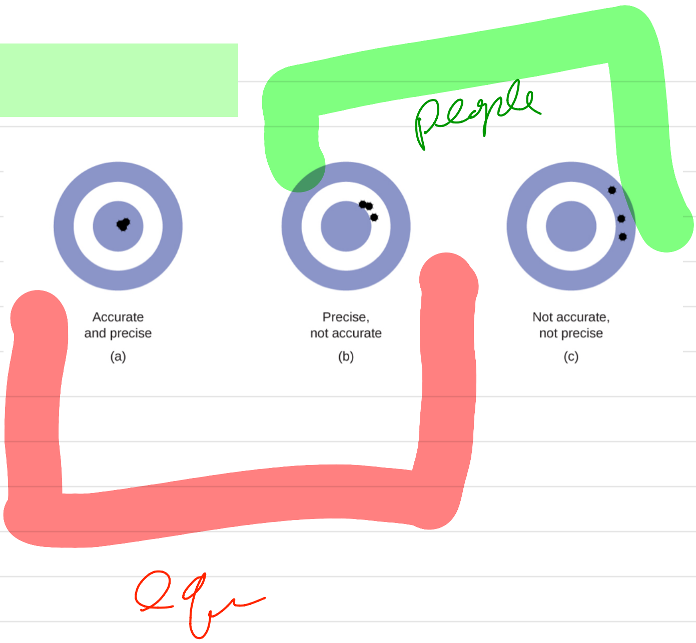
7 Prefixes

M	(mega)	1,000,000
K	(kilo)	1,000
d	(deci)	1/10
c	(centi)	1/100
m	(milli)	1/1000
μ	(micro)	1/1,000,000
n	(nano)	1/1,000,000,000

7 Fundamental Units

m	length	
Kg	mass	
s	time	
K	temp	
mol	amount ("batch")	6.02×10^{23}
A	current	
cd	luminosity	

Accuracy vs. Precision



Accuracy — ability to hit the bullseye

Precision — tight grouping… ability to hit the same spot every time… to not "spray" the data.

Determining and Expressing Uncertainty

$\% = \frac{(acc - exp)}{acc} \times 100$

In this course, there are basically three levels of questions regarding 'uncertainty' calculations:

(1) SINGLE NUMBER: Det'n the Sig Figs and Places of a single number

(2) SINGLE OPERATION: Det'n the Sig Figs and Places of the answer resulting from a (x/÷), or a (+/—), mathematical operation.

(3) MIXED OPERATION: Det'n the Sig Figs and Places of the answer resulting from a problem involving both (x/÷) and (+/—) operations.

120

12.030 — 5 s.f., 3 d.p.

$$\begin{array}{r} 12.030 \\ +\ 0.40 \\ \hline 12.430 \end{array}$$

$$\frac{12.030 + 0.40}{0.200}$$

CAVEAT!!! Determining SigFigs and Places is a CORE CONCEPT in this class… it is highly probably that every test will have at least one "SigFig" question on it… SigFig questions will also be on the final exam… and they will be graded on every lab report submitted… in short, SigFigs is not "going away."

Uncertainty: Single Number

The Box-and-Dot Method: How to count Sig Figs and Places

(1) Box from the first thur the last non-zero digits
(2) IF, and only if, you see a “dot”, draw a box around any TRAILING zeros
(3) all digits in the box(es) are significant, the others are not
… furthermore, the right-most boxed digit provides the “place” to which the number is precise

	SF	PLACE
3040	3	10's
3040.0	5	1 dp
304.0	4	1 dp
0.00304	3	5 dp
3.01×10^3	3	2 dp
100	1	100's
100.	3	1's

H T O 1 2 3
1111.000

The Box-and-Dot Method: A Simple Strategy for Counting Significant Figures

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When test time arrives many students struggle to correctly assign significant figures to numbers. They confuse (or forget altogether) simple rules previously memorized, making the rules difficult or impossible to apply under pressure situations.

This situation is largely remedied with the use of a visual approach I refer to as the box-and-dot method. The method uses the device of "boxing" significant figures based on two simple rules, then counting the number of digits in the box(es).

The three steps of the box-and-dot method are detailed below; additional examples show the method's ease of use.

Box-and-Dot Basics

The box-and-dot method consists of three simple steps for determining the significant figures in any real number (except zero). The steps must be followed explicitly.

Step 1

> Draw a box around all nonzero digits, beginning with the leftmost nonzero digit and ending with the rightmost nonzero digit in the number.

For example, drawing a box around the nonzero digits in the number 0.0123012300 gives 0.0[1230123]00. Any zero(s) trapped or "sandwiched" between nonzero digits will necessarily be included in the box.

For convenience, a digit or number surrounded by a box may be referred to as a "boxed" digit or "boxed" number, respectively.

Step 2

> If a dot is present, draw a box around any trailing zeros.

Continuing with the above example, a dot (decimal point) is present in the expression 0.0[1230123]00; therefore, trailing zeros[1] are boxed, which gives 0.0[1230123][00].

Step 2 uses the term ***dot*** in lieu of ***decimal point*** for reasons of brevity and ease of recall. "Box-and-decimal point method" possesses neither the pith nor lilt of "box-and-dot".

The position of a decimal point within a number is irrelevant—the only test for boxing trailing zeros is the mere presence of a dot.

Because method steps are followed explicitly, it is understood that trailing zeros should ***not*** be boxed when a dot is ***not*** present. In other words, draw a box around trailing zeros if and only if a dot is present.

Step 3

> Consider any and all boxed digits significant.

Boxed digits are significant, whereas digits that are not boxed are not significant. Continuing the example from Step 2, the expression 0.0[1230123][00] reveals nine digits surrounded by boxes. Therefore, there are nine significant figures. Specifically, the significant figures are: 1, 2, 3, 0, 1, 2, 3, 0, and 0.

Note that the box-and-dot method does not expressly address leading zeros.[2] There is no need. Leading zeros, which are never significant, always lie to the left of the box drawn in Step 1 and are therefore excluded from consideration as significant figures by virtue of the explicitness of Steps 1 and 2. Those steps provide criteria for boxing (and thus rendering significant) trapped and trailing zeros only, to the exclusion of all other zeros.

In general, at least one, but not more than two, boxes will be associated with any given number. Step 1 always requires that a box be drawn. Step 2 only allows for a (second) box to be drawn when two conditions are met: (i) a decimal point is present, and (ii) one or more trailing zeros are present.

Typically, none of the steps requires more than a few seconds to complete.

Worked Examples of the Box-and-Dot Method

Several examples follow, in the form of sample questions. For convenience, each of the three steps discussed above is represented with a chevron (»). The number prior to the first chevron in each example is the number for which significant figures are to be determined. To demonstrate the ease of use and visual nature of the box-and-dot method, little or no explanation is provided. Deriving the correct answer becomes second nature after working only a few problems.

Question 1

How many significant figures are in the number 123.01230?

Answer: 123.01230 » [123.0123]0 (nonzero digits are boxed) » [123.0123][0] (dot present, so the trailing zero is boxed) » eight numbers are boxed, therefore the number has eight significant figures.

Question 2

How many significant figures are in the number 12301230?

Answer: 12301230 » [1230123]0 » no dot is present (so the trailing zero is not boxed) » seven significant figures.

Question 3

How many significant figures are in the number 123.0123?

Answer: 123.0123 » [123.0123] » no trailing zeros » seven significant figures.

Question 4

How many significant figures are in the number 10100?

Answer: 10100 » [101]00 » no dot (so no trailing box) » three significant figures.

Sig Figs In Mathematical Operations: (x/÷) and (+/—)

	sf	pl
	x/÷	+/−
45.60	4	2dp
x 1.4	2	1dp

$$\begin{array}{r} 45.60 \\ \times\ 1.4 \\ \hline 63.84 \end{array}$$

63 → **64** (not 64.0)

$$\begin{array}{r} 45.60 \\ +\ 1.4 \\ \hline 47.00 \end{array}$$

→ **47.0**

REMEMBER…
- Trust the calculator's DIGITS, not it's NUMBER
- When determining the correct number: "Choose one: either SF or PLACES
- Apply the "weakest link" rule

(EX) Sig Figs In Mathematical Operations: (x/÷) and (+/—)

$$\begin{array}{r} 44.56 \\ \times\ \ 0.140 \\ \hline \end{array}$$

$$\begin{array}{r} 44.56 \\ +\ \ 0.140 \\ \hline \end{array}$$

	sf	pl
44.56	4	2 dp
× 0.140	3	3 dp

44.56 × 0.140 = 6.2384

ROUNDING: 6.24 ✓ / 6.23 ×

✓ 3 sf.
✓ rounded

44.56 + 0.140 = 44.700 → 44.70

✓ good to 2nd dp
✓

For Mixed Operations problems, use the "3-Box approach"

(EX) Mixed Operations: single problem with both +/– AND (x/÷) operations

How NOT do get the correct answer: punch all the numbers out on you calculator in one step

$$\% \text{Error} = \frac{98-94}{94} \times 100 = 4.25532$$

(pl) +/– ; x/÷ (SF) ; % = exact ; from calculator

How TO get the correct answer: use the 3-column approach

(1) write the entire equation to be solved in Column1	(2) solve only the add/sub (+/–), then write the entire new equation in Column2	(3) solve the mul/div (x/÷), then write the entire new equation in Column3
$\frac{98.0-94.0}{94.0} \times 100$	$\frac{4.0}{94} \times 100 =$	4.26

Remember the way to work mixed problems

(1) write the entire equation to be solved in Column1	(2) solve only the add/sub (+/–), then write the entire new equation in Column2	(3) solve the mul/div (x/÷), then write the entire new equation in Column3

Exact Numbers: Definitions and Examples

1. Conversions between units within the English System are exact.
~e.g. 12 in = 1 ft or 12 in/1 ft (In this conversion, 12 and 1 are both exact.)

2. Conversions between units within the Metric System are exact.
~e.g. 1 m = 100 cm or 1 m/100 cm (In this conversion, 1 and 100 are both exact.)

3. Conversions between English and Metric system are generally NOT exact. Exceptions will be pointed out to you.
~e.g. 1 in = 2.54 cm exactly (1 and 2.54 are both exact.)
~e.g. 454 g = 1 lb or 454 g/1 lb (454 has 3 sig. fig., but 1 is exact.)

4. “Per” means out of exactly one.
~e.g. 45 miles per hour means 45 mi = 1 hr or 45 mi/1 hr. (45 has 2 sig. fig. but 1 is exactly one.)

5. “Percent” means out of exactly one hundred.
~e.g. 25.9% means 25.9 out of exactly 100 or 25.9/100 (25.9 has 3 sig. fig., but 100 is exact.)

6. Counting numbers are exact. Sometimes it is hard to decide whether a number is a “counting number” or not. In most cases it would be obvious. Ask when in doubt.
~e.g. There are 5 students in the room. (5 would be an exact number because you cannot have a fraction of a student in the room.)
~e.g. subscripts in a formula, and coefficients in a balanced equation, are considered “counting numbers” and are exact

7. Mathematical constants are exact. The symbol is exact; however, the number 3.14 has only three significant figures, while 3.1416 has five. In a mathematical formula, such as V = (4/3)pr 3 , or P.E. = ½ mv 2 , the fractions are exact numbers.

8. The conversions between Celsius, Fahrenheit, and Kelvin temperatures are exact. This means the fractions (5/9 or 9/5) and the number 32 are exact. The number 273.15, in the Celsius to Kelvin temperature conversion, is also exact.

9. Speed of light in a vacuum is exact, and is equal to 299,792,458 m/s

3 3.0 3.0000 100. g

100
+ 1

101

100's → 100
1's → + 1

101

↓

100

Math Operations, Measured Numbers, and Exact Numbers

$$\begin{array}{r} 100 \\ \times\ \ 2 \\ \hline \end{array} \qquad \begin{array}{r} 100 \\ +\ \ 2 \\ \hline \end{array} \qquad \begin{array}{r} 100 \\ +\ \ 2 \\ \hline \end{array}$$

$$\begin{array}{r} 100 \\ \times\ \ 2 \\ \hline \underline{2}00 \end{array} \rightarrow \boxed{200}$$

1 s.f.

1 sf, 100's

1 sf, 1's

$$\begin{array}{r} 100 \\ +2 \\ \hline \underline{1}02 \end{array} \rightarrow \boxed{100}$$

100's place

Measured vs. Exact Numbers In Calculations

(x/÷, so count s.f.)

sf if 9 is measured	sf if 9 is exact
47.2 x 9	47.2 x 9

400 g

425 g

Dimensional Analysis (Factor Label Method)

It is extremely difficult to pass this class without being proficient — that is, being practiced and able to use with ease — dimensional analysis. (You have now been warned. Take heed.)

"Take heed of my instructions, little boy," said the old bearded man.
"My potion will only work for the one who wears the ring."

DA Basics: use of Conversion Factors

(EX) ¿If 2 loaves of bread cost $4.28, how much do 17 loaves cost?

DA Basics: 1-step (simple) unit conversions

(EX) ¿Convert 325 mL into L?

(EX) ¿Convert 0.325 L into KL (kiloliters)?

DA Basics: the all-important multistep (complex) conversion

(EX) ¿Convert 325 μL into KL (kiloliters)?

DA Basics: conversions of both numerator ("top") and denominator ("bottom")

(EX) ¿Express 26.2 mi/hr in km/min?

$$\frac{\square\ km}{min} = \frac{26.2\ mi}{hr}\left|\frac{1\ hr}{60\ min}\right|\frac{1760\ yd}{1\ mi}\left|\frac{36\ in}{1\ yd}\right|\frac{2.54\ cm}{1\ in}\left|\frac{1\ m}{100\ cm}\right|\frac{1\ Km}{1000\ m}$$

"do I get starting with..."

1000, 1

$$\frac{\square\ km}{min} = \frac{26.2\ mi}{1\ hr}\left|\frac{1\ hr}{60\ min}\right|\frac{1760\ yd}{1\ mi}\left|\frac{1\ m}{1.094\ yd}\right|\frac{1\ Km}{1000\ m} =$$

3sf, 3sf, 4sf

$= 0.702498477$ Km/min

$= \boxed{0.702\ Km/min}$

(EX) Dimensional Analysis

¿Your Ford Mustang has a 5.00 L engine. What is the engine size in units of “in³”?

$$\frac{\square\ in^3}{1} = \frac{5.00\ L}{1} \left| \frac{1000\ mL}{1\ L} \right| \frac{1\ cm^3}{1\ mL} \left| \frac{1^3\ in^3}{(2.54)^3\ cm^3} \right. = \boxed{305\ in^3}$$

16.387

Density

$$D = \frac{m}{V}$$

algebraically

$$\frac{\square\, g}{mL} =$$

dimensional analysis

(EX) ¿11.2 mL of a liquid was weighed by difference and found to have a mass of 9.5 g. What is the density?

algebraically

$$D = \frac{m}{V} = \frac{9.5}{11.2} = 0.85\ g/mL$$

dimensional analysis

$$\frac{\square\, g}{mL} = \frac{9.5\,g}{1} \,\Big|\, \frac{1}{11.2\,mL} = \frac{0.85\,g}{mL}$$

$$\frac{\square\, g}{mL} = \frac{9.50\,g}{1} \times \frac{1}{11.2\,mL} = \frac{0.848\,g}{mL}$$

(EX) Dimensional Analysis… REDUX: Density Question Previous Page:

¿The density of 55.64 g of a material is 21.4 g/mL. What is its volume?

$\left(\frac{21.4\,g}{1\,mL}\right)$ $\left(\frac{1\,mL}{21.4\,g}\right)$

The Hard Way

$D = \frac{m}{V}$ → $D \cdot V = m$ → $V = \frac{m}{D} = \frac{55.64\,g}{21.4\,g/mL} = 2.60\,mL$

algebraically

The Easy "No-algebra" Way

$\frac{\square\, g}{mL} =$ $\frac{\square\, mL}{1} = \frac{1\,mL}{21.4\,g} \Big| \frac{55.64\,g}{1} = 2.60\,mL$

dimensional analysis

—— answer next page ——

(EX) Dimensional Analysis… REDUX: Density Question Previous Page:
¿The density of 55.64 g of a material is 21.4 g/mL. What is its volume?

The Hard Way

$$D = \frac{m}{V}$$

algebraically

$D = \frac{m}{V} \rightarrow V = \frac{m}{D} = \frac{55.64\,g}{21.4\,g/mL} = \boxed{2.60\,mL}$

① write equation ② re-arrange equation ③ plug in numbers, & solve

The Easy “No-algebra” Way

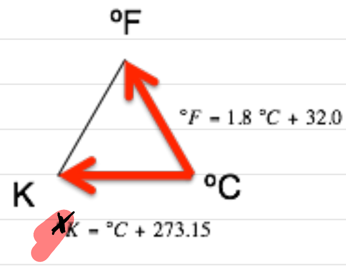
$$\frac{\square\, g}{mL} =$$

dimensional analysis

$$\frac{\square\, mL}{1} = \frac{1\, mL}{21.4\, g} \bullet \frac{55.64\, g}{1} = 2.60\, mL$$

GOOD NEWS: DA will work on equations that are all (x/÷) …
which probably turns out to be over 90%
of the equations you will use in this course.

Temperature Scales



$$^{\circ}F = 1.8\ ^{\circ}C + 32.0$$

$$K = \ ^{\circ}C + 273.15$$

exact numbers

BAD NEWS: can NOT use dimensional analysis for temperature conversions b/c of the presence of a "+/–" mathematical operator … these must be solved algebraically.

GOOD NEWS: this is the only equation this semester for which dimensional analysis does not work; all remaining equations are all "×/÷".